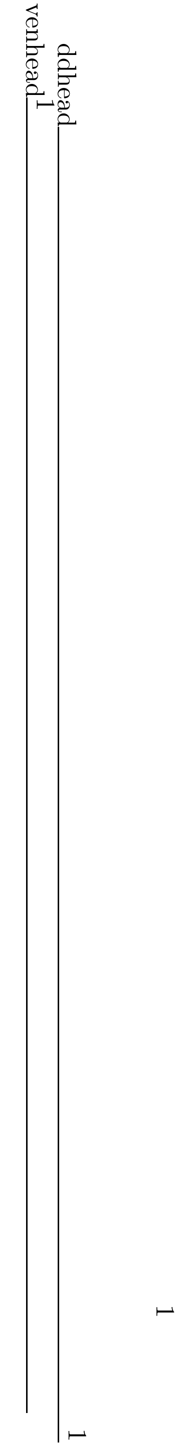
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EFFECTIVE AVERAGE ACTION AND NONPERTURBATIVE RENORMALIZATION GROUP EQUATION IN HIGHER DERIVATIVE QUANTUM GRAVITY

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We study the exact renormalization group (RG) in R^2-gravity in the effective average action formalism using the background field method. The truncated evolution equation (where truncation is made to low-derivatives functionals space) for such a theory in a de Sitter background leads to a set of nonperturbative RG equations for cosmological and gravitational coupling constants. The gauge dependence problem is solved by working in the physical Landau-DeWitt gauge corresponding to gauge-fixing independent effective action. Approximate solution of nonperturbative RG equations reveals the appearence of antiscreening or screening behaviour of Newtonian coupling, depending on the higher-derivatives coupling constants. The existence of unstable UV fixed points is also mentioned.

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It is well-known that Einstein quantum gravity (QG) is not renormalizable [1]. There are QG models which represent extensions of Einstein gravity. One of them, the so-called R^2-gravity (see [2] for an introduction and review) is multiplicatively renormalizable. However, it is most probably a non-unitary theory, at least in the perturbative approach. In the situation when consistent quantum gravity (QG) is unknown it is quite reasonable to study the existing gravitational theories as effective theories. This gives one the possibility of estimating QG manifestations at low energy scales.

In such a way one can reduce QG to a simpler theory described by some type of scalar Lagrangian [3, 4]. Those models are useful for describing QG in the far infrared domain (at large distances) [3, 4].

One can consider another approach. Let us take non-renormalizable Einstein gravity and work with it as with a usual non-renormalizable effective field theory. Then the calculation of quantum corrections is still possible. In such a way, quantum corrections to the Newtonian coupling constant and to the Newtonian potential have been estimated [5].

Finally, one can apply the exact RG [6] in the study of non-renormalizable theories. There has been recently much activity in studying different theories (mainly scalar ones) using the non-perturbative RG

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(for a list of recent papers see [7] and references therein). Using an average effective action and the background field method, a non-perturbative RG study of Einstein quantum gravity was recently presented [8]. The RG equation (or evolution equation) governing the evolution of the effective action from a scale $\Lambda_{\rm cut-off}$ where theory is well-defined to smaller scales $k < \Lambda_{\rm cut-off}$ has been constructed in Ref. [8]. Its gauge dependence has been investigated in [9]. It has been shown [9] that in the physical gauge (the Landau-DeWitt gauge) the Newtonian coupling constant shows antiscreening.

In the present paper we formulate the non-perturbative RG equation (evolution equation) in higher derivative QG (for a review and list of references see [2]). It is widely known that such a theory, being multiplicatively renormalizable and asymptotically free, has a perturbatively non-unitary S-matrix. Nevertheless, such a theory has a lot of applications. For example, it may lead to more or less succesful inflation [10]. Attempts to construct supersymmetric generalizations of R^2-gravity have been recently made [11, 12].

We consider higher-derivative QG as an effective theory, so issues of renormalizability or (non)unitarity are not important for us. We adopt the formalism of Refs. [8, 9] in such a theory and construct the scale-dependent gravitational average action $\Gamma_k[g_{\mu\nu}]$ in the background field formalism. A truncated evolution equation is obtained. We work in the physical gauge corresponding to a gauge-fixing independent effective action. Note that we make truncation of the average effective action to the space of low derivatives functionals only. Even in such a simplified variant (where higher-derivative couplings may be considered as free parameters) the calculation of nonperturbative RG equations is very complicated.

The paper is written as follows. In the next section we a give very brief review of the evolution equation (for more details see [8, 9]) and our truncation. Sec. 3 is devoted to the calculation of the one-loop effective action in R^2-gravity in the De Sitter background. Such an evaluation is presented in two cases: (a) a convenient effective action in a one-parameter-dependent gauge and (b) a gauge-fixing-independent effective action. The results of the above calculation are used to obtain the effective average action in the background field formalism for these two cases. (In other words, we obtain the r.h.s. of the evolution equation in the De Sitter background). In Sec. 4 we perform an explicit truncation of the evolution equation and obtain non-perturbative RG equations for the gravitational and cosmological coupling constants. In order to avoid the gauge dependence problem we work there in the gauge-fixing independent EA formalism (see [13] for an introduction). The critical points of the RG equations for the Newtonian and cosmological couplings and running Newtonian coupling are discussed in Sec. 5. Finally, some remarks are given in the conclusion.

estartsectionsection10pt-3.5ex plus -1ex minus -.2ex2.3ex plus .2exEvolution equation for average effective action

We will start from a short introduction to the average effective action approach in quantum gravity. We follow mainly Ref. [8] where more details are presented.

The basic elements of the approach are:

1. The background field method [2] implying that

$$g_{\mu\nu} = \bar g_{\mu\nu} + h_{\mu\nu} \tag{0.1}$$

where $\bar g_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ is a quantum fluctuation.

2. A scale-dependent generating functional for the connected Green functions

$$\begin{aligned} &W_k[t^{\mu\nu},\sigma^\mu,\bar\sigma_\mu;\beta^{\mu\nu},\tau_\mu;\bar g_{\mu\nu}] \\ &= \int Dh_{\mu\nu} DC^\mu D\bar C^\nu \exp\{-S[\bar g + h] - S_{\rm gf}[h;\bar g] \\ &- S_{gh}[h,C,\bar C;\bar g] - \Delta_k S[h,C,\bar C;\bar g] - S_{\rm source}\}. \end{aligned} \tag{0.2}$$

Let us give a description of the quantities which enter into Eq. (2.2). $S[\bar g + h]$ is the classical action of gravity theory under discussion; $S_{\rm gf}$ denotes the gauge-fixing term. As we will be interested in R^2-gravity, we suppose that $S_{\rm gf}$ may be of the fourth order in the derivatives. The set of ghosts $C, \bar C$ includes all ghosts in the theory (in R^2-gravity we have an extra ghost, the so-called third ghost). Finally, $\Delta_k S$ is the infrared (IR) cut-off for the gravitational field and ghosts. An introduction to the present formalism of studying the average effective action has been presented in all detail in [8], so we will not present more details of it here. $S_{\rm source}$ in (2.2) is the standard action describing the coupling of the gravitational field and ghosts with the sources $t^{\mu\nu}$, σ^μ, $\bar\sigma_\mu$.

Performing a Legendre transform of W_k to get the average effective action $\Gamma_k[g,\bar g]$, we can obtain the truncated evolution equation for Γ_k (see [8] for more details)

$$\begin{aligned} \partial_t \Gamma_k[g,\bar g] &= \frac12 \,\mathrm{Tr}\left[\left(A\Gamma_k^{(2)}[g,\bar g] + R_k^{\rm grav}[\bar g]\right)^{-1} \partial_t R_k^{\rm grav}[\bar g]\right] \\ &- \sum_i c_i \,\mathrm{Tr}\left[\left(-M_i[g,\bar g] + R_{ki}^{\rm gh}[\bar g]\right)\partial_t R_{ki}^{\rm gh}[\bar g]\right] \end{aligned} \tag{0.3}$$

where $t = \ln k$, k is the nonzero momentum scale, A is some constant which depends on the model under discussion (for Einstein gravity $A = \kappa^2$), R_k are cut-offs, c_i are the weights for ghosts. For the case of R^2-gravity we have the Fadeev-Popov ghost with $c_{\rm FP} = 1$ and the third ghost with weight $c_{\rm TG} = 1/2$, and, of course, $M_{\rm FP}$ and $M_{\rm TG}$ are usually known. $\bar g_{\mu\nu}$ is the background metric and $g_{\mu\nu} = \bar g_{\mu\nu} + h_{\mu\nu}$ where $h_{\mu\nu}$ is the quantum field. $\Gamma_k^{(2)}$ is the Hessian of $\Gamma_k[g,\bar g]$ with respect to $g_{\mu\nu}$ at fixed $\bar g_{\mu\nu}$.

The next step is to specify the truncated evolution equation for the theory under study. We start from the

R^2-gravity (3.1) at the UV scale $\Lambda_{\mathrm{cut-off}}$ and evolve it down to smaller scales $k << \Lambda_{\mathrm{cut-off}}$. We use the truncation where the coupling constants are replaced by the k-dependent functions (see (3.1))

$$\kappa^2 \to Z_{Nk}^{-1}\kappa^2, \qquad \frac{1}{f^2} \to Z_{Nk}\frac{1}{f^2},$$
$$\frac{1}{\nu^2} \to Z_{Nk}\frac{1}{\nu^2}, \qquad \Lambda \to \bar{\lambda}_k. \tag{0.4}$$

Note that we do not write explicitly the k-dependence for the higher-derivatives coupling constants because we will be restricted here only to lower-derivative terms (i.e. up to the linear curvature term). Then, in such an approach (subreduction of the full set of RG equations), the higher-derivative coupling constants may be considered as free parameters of the theory.

Then, closely following the arguments of Ref. [8], we get (keeping only low-derivative terms)

$$\Gamma_k[g,g] = 2\kappa^2 Z_{Nk}\int d^4x\sqrt{g}\left[-R(g) + 2\bar{\lambda}_k\right]. \tag{0.5}$$

The ghost term disappears after choosing $\bar{g}_{\mu\nu} = g_{\mu\nu}$. Projecting the evolution equation on the space with low-derivatives terms, one gets the left-hand side of the truncated evolution equation (2.3) as follows

$$\partial_t\Gamma_k[g,g] = 2\kappa^2\int d^4x\sqrt{g}[-R(g)\partial_t Z_{Nk} + 2\partial_t(Z_{Nk}\bar{\lambda}_k)]. \tag{0.6}$$

The initial conditions for Z_{Nk}, λ_k are choosen in the same way as in [8]. The right-hand side of the truncated evolution equation (2.3) will be defined in the next section, following similar one-loop arguments. We have to note only that, unlike the Einstein gravity, the projectors should include the coupling constants. We do not give more details as they are very similar to those discussed in [8].

estartsectionsection10pt-3.5ex plus -1ex minus -.2ex2.3ex plus .2exOne-loop effective action and effective average action in R^2-gravity

In this section we study the one-loop effective action and the average effective action in higher-derivative quantum gravity (for a review see [2] and references therein).

The classical action in Euclidean notations has the following form:

$$S = \int d^4x\sqrt{g}\left\{\epsilon R^*R^* + \frac{1}{2f^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - \frac{1}{6\nu^2}R^2 - 2\kappa^2 R + 4\kappa^2\Lambda\right\} \tag{0.7}$$

where $R^*R^* = \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\lambda\rho\gamma\delta}R^{\lambda\rho}_{\mu\nu}R^{\gamma\delta}_{\alpha\beta}$, $C_{\mu\nu\alpha\beta}$ is the Weyl tensor, $\kappa^{-2} = 32\pi\bar{G}$ is the Newtonian coupling constant, ϵ, f^2, ν^2 are the gravitational coupling constants related to the higher-derivative terms in (3.1). It is quite well-known that the theory with the action (3.1) is multiplicatively renormalizable and asymptotically free (see [2] for a review).

Our first purpose will be to calculate the one-loop effective action for the theory with action (3.1) on de Sitter background:

$$R_{\mu\nu\alpha\beta} = \frac{1}{12}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)R,$$
$$R_{\mu\nu} = \frac{1}{4}g_{\mu\nu}R. \tag{0.8}$$

We work in the usual background field method [2], where the background field is given by the de Sitter metric,

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} + h_{\mu\nu} \tag{0.9}$$

and $h_{\mu\nu}$ is the quantum gravitational field.

We will be interested in the calculation of the effective action in a parameter-dependent gauge. Then, even in the case of Einstein gravity [14, 15] it is known that one has to make a change of the quantum fields:

$$h_{\mu\nu} = \bar{h}^{\perp}_{\mu\nu} + 2\nabla_{(\mu}\xi_{\nu)} + \frac{1}{4}g_{\mu\nu}h - \frac{1}{4}g_{\mu\nu}\Box\,\sigma,$$
$$h = h^{\mu\nu}g_{\mu\nu}, \qquad \nabla^\mu\xi^\perp_\mu = 0, \qquad \nabla^\mu h^\perp_{\mu\nu} = 0,$$
$$\xi_\mu = \xi^\perp_\mu + \frac{1}{2}\nabla_\mu\sigma, \qquad \bar{h}^\perp_{\mu\nu}g^{\mu\nu} = 0. \tag{0.10}$$

Clearly the transformation (3.4) induces a nontrivial Jacobian which should be taken into account in calculating the one-loop effective action.

The second variation of the classical action (3.1) in terms of the variables (3.4) is written as follows [16]:

$$\begin{aligned}\delta^2 S = \int d^4x\sqrt{g}\Bigg\{&\frac{1}{4f^2}\bar{h}^\perp \times\left[\Delta_2\Big(m_2^2 + \frac{f^2+\nu^2}{3\nu^2}R\Big)\Delta_2\Big(\frac{R}{6}\Big) + \frac{1}{2}m_2^2(R-4\Lambda)\right]\bar{h}^\perp\\ &- \frac{3}{32\nu^2}\Bigg[(h-\Box\,\sigma)\Bigg(\Delta_0(m_0^2)\Delta_0(-R/3) + \frac{1}{3}m_0^2(R-4\Lambda)\Bigg)(h-\Box\,\sigma)\\ &- \frac{2}{3}m_0^2(R-4\Lambda)(h-\Box\,\sigma)\Delta_0(0)\sigma\\ &- \frac{2}{3}m_0^2(R-4\Lambda)\sigma\Delta_0(0)\Delta_0(-R/2)\sigma\Bigg]\\ &+ 2\kappa^2(R-4\Lambda)\epsilon^\perp\Delta_1(-R/4)\epsilon^\perp\Bigg\}\end{aligned} \tag{0.11}$$

where $m_2^2 = 2\kappa^2 f^2$, $m_0^2 = 2\kappa^2\nu^2$. In accordance with [14] the constrained differential operators are introduced:

$$\Delta_0(X)\phi = (-\Box + X)\phi,$$
$$\Delta_{1\mu\nu}(X)\xi^{\nu\perp} = (-\Box_{\mu\nu} + g_{\mu\nu}X)\xi^{\nu\perp},$$
$$\Delta^{\mu\nu}_{2\alpha\beta}(X)\bar{h}^\perp_{\mu\nu} = (-\Box^{\mu\nu}_{\alpha\beta} + \delta^\mu_\alpha\delta^\nu_\beta X)\bar{h}^\perp_{\mu\nu}. \tag{0.12}$$

At the next step one can choose the gauge fixing term as follows:

$$S_{\rm GF} = \frac{1}{2}\int d^4x\sqrt{g}\chi_\mu H^{\mu\nu}\chi_\nu \tag{0.13}$$

where

$$\chi_\mu = -2\nabla_\nu\left\{h^\nu_\mu - \frac{1}{4}(1-K)\delta^\nu_\mu h\right\}, \qquad H^{\mu\nu} = \frac{1}{4\alpha^2}\left\{g^{\mu\nu}\left(-\Box + \frac{R}{4}\right)\right\} \tag{0.14}$$

and α^2, K are gauge parameters.

The general expression for the one-loop EA is given by

$$\Gamma = S + \frac{1}{2}\ln\det(\delta^2 S + S_{\rm GF}) - \frac{1}{2}\ln\det H^{\mu\nu} - \ln\det M^{\mu\nu} \tag{0.15}$$

where the standard ghost operator $M^{\mu\nu}$ is calculated with the help of χ_μ as follows:

$$M_{\mu\nu} = 2\left\{g_{\mu\nu}\left(-\Box - \frac{R}{4}\right) + \frac{1}{2}(K-1)\nabla_\mu\nabla_\nu\right\} \tag{0.16}$$

and the third ghost operator $H^{\mu\nu}$ is given in (3.8). Note that when the operators $H^{\mu\nu}$, $M^{\mu\nu}$ are written, the properties of the de Sitter background (3.2) are taken into account.

We follow in the evaluation of the one-loop EA the results of Ref. [16] where this calculation was performed in a more complicated six-parametric gauge. For our purposes, the only case $\alpha^2 = 0$, K being arbitrary (a Landau-DeWitt type gauge), will be of interest.

Taking into account the ghost operators and the Jacobian of the variables change (3.4), we obtain [16]

$$\begin{aligned}\Gamma^{(1)} &= \frac{1}{2}\ln\det\left[\Delta_2\left(\frac{R}{6}\right)\Delta_2\left(m_2^2 + \frac{f^2+\nu^2}{3\nu^2}R\right) + \frac{1}{2}m_2^2(R-4\Lambda)\right] - \frac{1}{2}\ln\det\Delta_1\left(-\frac{R}{4}\right)\\ &+ \frac{1}{2}\ln\det\left[\Delta_0^2\left(\frac{R}{K-3}\right)\Delta_0(m^2) + m_0^2\frac{K^2-3}{(K-3)^2}\Delta_0\left(\frac{R}{K^2-3}\right)(4\Lambda - R)\right] - \ln\det\Delta_0\left(\frac{R}{K-3}\right)\end{aligned} \tag{0.17}$$

Let us present Eq. (3.11) in the form of gravitational and ghost contributions:

$$\begin{aligned}\Gamma^{(1)}_{\rm grav} &= \frac{1}{2}\ln\det\left[\Delta_2\left(\frac{R}{6}\right)\Delta_2\left(m_2^2 + \frac{f^2+\nu^2}{3\nu^2}R\right) + \frac{1}{2}m_2^2(R-4\Lambda)\right] + \frac{1}{2}\ln\det\Delta_1\left(-\frac{R}{4}\right) + \frac{1}{2}\ln\det\Delta_1\left(\frac{R}{4}\right)\\ &+ \frac{1}{2}\ln\det\left[\Delta_0^2\left(\frac{R}{K-3}\right)\Delta_0(m_0)^2 + m_0^2\frac{K^2-3}{(K-3)^2}\Delta_0\left(\frac{R}{K^2-3}\right)(4\Lambda - R)\right] + \frac{1}{2}\ln\det\Delta_0(0),\end{aligned} \tag{0.18}$$

$$\begin{aligned}\Gamma^{(1)}_{\rm ghost} &= -\frac{1}{2}\ln\det H^{\mu\nu} - \ln\det M^{\mu\nu}\\ &= -\frac{1}{2}\ln\left[\det\Delta_1\left(\frac{R}{4}\right)\det\Delta_0(0)\right] - \ln\left[\det\Delta_1\left(-\frac{R}{4}\right)\det\Delta_0\left(\frac{R}{K-3}\right)\right]\end{aligned} \tag{0.19}$$

where $\Gamma^{(1)} = \Gamma^{(1)}_{\rm grav} + \Gamma^{(1)}_{\rm ghost}$. The gauge dependence of the one-loop effective action is clearly seen in Eqs. (3.11)–(3.13).

In order to avoid the explicit gauge dependence one can work with the gauge-fixing independent EA (for an introduction see [2, 13]). An explicit calculation has been done in Ref. [16] with the following result:

$$\begin{aligned}\Gamma^{(1){\rm GFI}}_{\rm grav} &= \frac{1}{2}\ln\det\left[\Delta_2\left(\frac{R}{6}\right)\Delta_2\left(m_2^2 + \frac{f^2+\nu^2}{3\nu^2}R\right) + \frac{1}{2K}m_2^2(R-4\Lambda)\right] + \frac{1}{2}\ln\det\Delta_1\left(-\frac{R}{4}\right)\\ &+ \frac{1}{2}\ln\det\Delta_1\left(\frac{R}{4}\right) + \frac{1}{2}\ln\det\left[\Delta_0\left(\frac{R}{K-3}\right)\Delta_0\left(m_0^2\right) - \frac{1}{K-3}m_0^2\left(R-4\Lambda\right)\right]\\ &+ \frac{1}{2}\ln\det\Delta_0(0) + \frac{1}{2}\ln\det\Delta_0\left(\frac{R}{K-3}\right)\end{aligned} \tag{0.20}$$

where the parameter K is fixed: $K = 3f^2/(f^2+2\nu^2)$. The ghost contribution in the gauge-fixing independent EA formalism is given again by (3.13) with K fixed as above. Hence, we also found the one-loop gauge-fixing independent EA. The use of such EA solves the problem of gauge dependence of the EA (for a discussion of the dependence of the gauge-fixing independent EA on the metric in the space of fields in quantum gravity see [17]).

Note that the gauge-fixing independent one-loop EA in Einstein quantum gravity in a constant-curvature space (a background like the de Sitter space) has been discussed in Ref. [18], (see [2] for a review).

Our final goal is related to a study of the truncated evolution equation. A necessary step in such a study is the expansion of the average effective action in powers of the curvature. Effectively, one should use the one-loop effective action in such an expansion.

However, the transition to constrained differential operators in accordance with (3.4) introduces additional zero modes. This leads to a wrong answer when we expand the determinants of the constrained operators in powers of the curvature. Therefore it is better to represent the effective action in terms of unconstrained operators. It could be done with the help of the following relations [14]:

$$\begin{aligned}
&\det\Delta_V(X)\equiv\det(-\Box+X)_V=\det\Delta_1(X)\det\Delta_0\left(X-\frac{R}{4}\right),\\
&\det\Delta_T(X)\equiv\det\left(-\Box+X\right)_T=\det\Delta_2(X)\det\Delta_1(X-\tfrac{5}{12}R)\det\Delta_0(X-\tfrac{2}{3}R).
\end{aligned}\tag{0.21}$$

The operators from the left-hand side are unconstrained. Note also that in order to apply the relations (3.15) we should also rewrite the higher-derivative operators in terms of low-derivative (second order) ones.

For simplicity, we consider below only the one-loop EA (3.14), (3.13). This EA actually describes two cases: for $K=1$ it coincides with the standard EA (3.12), (3.13) in the gauge $K=1$ and for $K=3f^2/(f^2+2\nu^2)$ it describes the gauge-fixing-independent EA.

First of all, we rewrite the ghost contribution (3.13) in terms of unconstrained operators:

$$\Gamma^{(1)}_{\text{ghost}}=-\frac{1}{2}\ln\det\Delta_V\left(\frac{R}{4}\right)-\ln\left[\det\Delta_V\left(-\frac{R}{4}\right)\det\Delta_0\left(\frac{R}{K-3}\right)\overset{-1}{\det}\Delta_0\left(-\frac{R}{2}\right)\right]\tag{0.22}$$

Hence the ghost part is expressed in terms of unconstrained operators.

For the gravitational part we get

$$\begin{aligned}
\Gamma^{(1)GFI}_{\text{grav}}=&\frac{1}{2}\ln\det\left\{\frac{\Delta_T\left(\frac{b+\sqrt{b^2-4c}}{2}\right)}{\Delta_V\left(\frac{b+\sqrt{b^2-4c}}{2}-\frac{5}{12}R\right)}\right\}+\frac{1}{2}\ln\det\left\{\frac{\Delta_T\left(\frac{b-\sqrt{b^2-4c}}{2}\right)}{\Delta_V\left(\frac{b-\sqrt{b^2-4c}}{2}-\frac{5}{12}R\right)}\right\}\\
&+\frac{1}{2}\ln\det\left\{\frac{\Delta_V(-R/4)}{\Delta_0(-R/2)}\right\}+\frac{1}{2}\ln\det\Delta_V\left(\frac{R}{4}\right)\\
&+\frac{1}{2}\ln\det\left[\Delta_0\left(\frac{R}{K-3}\right)\Delta_0(m_0^2)-\frac{1}{K-3}m_0^2\left(R-4\Lambda\right)\right]+\frac{1}{2}\ln\det\Delta_0\left(\frac{R}{K-3}\right)
\end{aligned}\tag{0.23}$$

where

$$b=\frac{R}{6}+m_2^2+\frac{f^2+\nu^2}{3\nu^2}R,\qquad c=\frac{R}{6}\left(m_2^2+\frac{f^2+\nu^2}{3\nu^2}R\right)+\frac{1}{2K}m_2^2\left(R-4\Lambda\right)$$

Recall that for $K=1$, (3.16) plus (3.17) gives the standard one-loop EA in the gauge $K=1$. However, now this EA is expressed in terms of unconstrained differential operators.

Now we can write the average effective action in the theory. First of all, to write the evolution equation we have to include the cut-off term $\Delta_k S$. In other words, in the calculation of $W_k=\ln Z_k$ in the exponent of the path integrand we have to consider not only $\Gamma^{(2)}_{k\ \text{grav}}$, $\Gamma^{(2)}_{k\ \text{gh}}$ and the ghost term, but also $\Delta_k S$.

The coefficients Z_k^{grav} and Z_k^{gh} should be chosen so that the kinetic and cut-off terms combine to $-\Box+k^2R^{(0)}(-\Box/k^2)$ for every degree of freedom. Here $R^{(0)}$ is a dimensionless cut-off function. As in the case of pure Einstein gravity [8, 9], all renormalization effects of ghosts are neglected.

Hence the effective average action may be written in the form (see Eqs. .(3.16) and (3.17))

$$\begin{aligned}
\Gamma_k[g,g]=&\frac{1}{2}\,\mathrm{Tr}_T\ln\left\{Z_{Nk}\left(-\Box+\frac{b+\sqrt{b^2-4c}}{2}+k^2R^{(0)}\right)\right\}\\
&+\frac{1}{2}\,\mathrm{Tr}_T\ln\left\{Z_{Nk}\left(-\Box+\frac{b-\sqrt{b^2-4c}}{2}+k^2R^{(0)}\right)\right\}\\
&-\frac{1}{2}\,\mathrm{Tr}_V\ln\left\{Z_{Nk}\left(-\Box+\frac{b+\sqrt{b^2-4c}}{2}-\frac{5}{12}R+k^2R^{(0)}\right)\right\}
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{2}\,\mathrm{Tr}_V \ln\left\{ Z_{Nk}\left(-\Box + \frac{b-\sqrt{b^2-4c}}{2} - \frac{5}{12}R + k^2 R^{(0)}\right)\right\} + \frac{1}{2}\,\mathrm{Tr}_V \ln\left\{ Z_{Nk}\left(-\Box - \frac{R}{4} + k^2 R^{(0)}\right)\right\} \\
&- \frac{1}{2}\,\mathrm{Tr}_0 \ln\left\{ Z_{Nk}\left(-\Box - \frac{R}{2} + k^2 R^{(0)}\right)\right\} + \frac{1}{2}\,\mathrm{Tr}_V \ln\left\{ Z_{Nk}\left(-\Box + \frac{R}{4} + k^2 R^{(0)}\right)\right\} \\
&+ \frac{1}{2}\,\mathrm{Tr}_0 \ln\left\{ Z_{Nk}\left(-\Box + \frac{A_1+\sqrt{A_1^2-4B_1}}{2} + k^2 R^{(0)}\right)\right\} \\
&+ \frac{1}{2}\,\mathrm{Tr}_0 \ln\left\{ Z_{Nk}\left(-\Box + \frac{A_1-\sqrt{A_1^2-4B_1}}{2} + k^2 R^{(0)}\right)\right\} \\
&- \frac{1}{2}\,\mathrm{Tr}_V \ln\left\{ \left(-\Box + \frac{R}{4} + k^2 R^{(0)}\right)\right\} - \mathrm{Tr}_V \ln\left\{ \left(-\Box - \frac{R}{4} + k^2 R^{(0)}\right)\right\} \\
&- \mathrm{Tr}_0 \ln\left\{ \left(-\Box + \frac{R}{K-3} + k^2 R^{(0)}\right)\right\} + \mathrm{Tr}_0 \ln\left\{ \left(-\Box - \frac{R}{2} + k^2 R^{(0)}\right)\right\} \qquad (0.24)
\end{aligned}
$$

where

$$
A_1 = \frac{R}{K-3} + m_0^2, \qquad B_1 = \frac{4m_0^2\Lambda}{K-3}
$$

and Λ should be replaced by $\bar{\lambda}_k$ in (3.18). Thus we have got the average effective action.

estartsectionsection10pt-3.5ex plus -1ex minus -.2ex2.3ex plus .2exEvolution equations for the Newtonian and cosmological constants

In this section we write down the renormalization group equation (2.3) for the action (3.1). The l.h.s. of the truncated evolution equation is given by (2.6), where we have projected the evolution equation on the space with low derivatives.

Now we want to find the r.h.s. of the evolution equation. To this end, we differentiate Eq. (3.18) with respect to t. Then we expand the operators in (3.18) in the curvature R because we are only interested in terms of the order $\int d^4x\sqrt{g}$ and $\int d^4x\sqrt{g}R$. We also have to expand some functions of R inside the operators that appear in the first four terms and in the eighth and ninth terms in Eq. (3.18) up to linear terms in R.

Let us take the first two terms in Eq. (3.18) and represent

$$
f_{1,2}(R) = \frac{b \pm \sqrt{b^2-4c}}{2} \qquad (0.25)
$$

where b and c are given by (3.17). These functions may be written as

$$
f_{1,2}(R) = \frac{1}{2}\left[\frac{R}{6} + A \pm \sqrt{BR^2+CR+D}\right] \qquad (0.26)
$$

where A, B, C, D depend on f, ν, Λ through b, c, m_0^2 and m_2^2. Expanding (4.2) up to terms linear in curvature

$$
f_{1,2} = \frac{1}{2}\left[\frac{R}{6} + A \pm \left(D^{1/2} + \frac{1}{2}D^{-1/2}\left(BR^2+CR\right) + ...\right)\right] \simeq \frac{1}{2}\left[A \pm D^{1/2} + R\left(\frac{1}{6} \pm \frac{1}{2}CD^{-1/2}\right)\right], \qquad (0.27)
$$

we can write

$$
f_1 = \alpha_1 R + \alpha_2, \qquad f_2 = \beta_1 R + \beta_2
$$

where

$$
\begin{aligned}
\alpha_1,\ \beta_1 &= \frac{1}{4}\left\{\frac{1}{3} \pm CD^{-1/2}\right) = \frac{1}{12} + \frac{f^2+\nu^2}{6\nu^2} \pm \frac{1}{2}\left(\frac{f^2+\nu^2}{3\nu^2} - \frac{K+6}{6K}\right) m_2^2 \left[m_2^4 + \frac{8\Lambda m_2^2}{K}\right]^{-1/2}; \\
\alpha_2,\ \beta_2 &= \frac{1}{2}\left(A \pm D^{1/2}\right) = \frac{1}{2}m_2^2 \pm \frac{1}{2}\left[m_2^4 + \frac{8\Lambda m_2^2}{K}\right]^{1/2} \qquad (0.28)
\end{aligned}
$$

Here we have replaced the values of C and D.

The functions f_1 and f_2 are the same for the third and fourth terms in (3.18). In a similar manner we obtain for the eighth and ninth terms in (3.18) the following values:

$$
f_3 = \gamma_1 R + \gamma_2, \qquad f_4 = \delta_1 R + \delta_2
$$

with

$$\gamma_1,\ \delta_1 = \frac{1}{2(K-3)} \pm \frac{m_0^2}{2(K-3)}\left(m_0^4 - \frac{16m_0^2\Lambda}{K}\right)^{-1/2}, \qquad \gamma_2,\ \delta_2 = \frac{1}{2}m_0^2 \pm \frac{1}{2}\left(m_0^4 - \frac{16m_0^2\Lambda}{K-3}\right)^{1/2}. \tag{0.29}$$

After this expansion we can expand the operators in (3.18) linearly in R. Let us introduce the notation

$$\Delta_i^{-1}\left(\alpha R + \lambda + k^2R^{(0)}\right) = \Delta_i^{-1}\left(\lambda + k^2R^{(0)}\right) - \Delta_i^{-2}\left(\lambda + k^2R^{(0)}\right)\alpha R + 0(R^2) \tag{0.30}$$

where a takes the values $\alpha_1,\ \beta_1,\ \gamma_1,\ \delta_1$ and λ takes the values $\alpha_2,\ \beta_2,\ \gamma_2,\ \delta_2$, given by (4.4), (4.5). For a more compact notation let us introduce

$$\Delta_{i\lambda}(z) = \Delta_i\left(\lambda + k^2R^0(z)\right), \tag{0.31}$$

$$N(z) = \frac{\partial_t[Z_{Nk}k^2R^{(0)}(z)}{Z_{Nk}}, \qquad N_0(z) = \partial_t\left[k^2R^{(0)}(z)\right]. \tag{0.32}$$

Here the variable z replaces $-\Box/k^2$. Note that we use as a cut-off the same function as in Ref. [8]: $R^{(0)}(z) = z/\mathrm{Exp}[z]-1)$. The above steps then lead to

$$\begin{aligned}
\partial_t\Gamma_k[g,g] = {} & \frac{1}{2}\mathrm{Tr}_T\left[N\Delta_{T\alpha_2}^{-1}\right] + \frac{1}{2}\mathrm{Tr}_T\left[N\Delta_{T\beta_2}^{-1}\right] - \frac{1}{2}\mathrm{Tr}_V\left[N\Delta_{V\alpha_2}^{-1}\right] - \frac{1}{2}\mathrm{Tr}_V\left[N\Delta_{V\beta_2}^{-1}\right] + \mathrm{Tr}_V\left[N\Delta_{V0}^{-1}\right] \\
& - \frac{1}{2}\mathrm{Tr}_S\left[N\Delta_{S0}^{-1}\right] + \frac{1}{2}\mathrm{Tr}_S\left[N\Delta_{S\gamma_2}^{-1}\right] + \frac{1}{2}\mathrm{Tr}_S\left[N\Delta_{S\delta_2}^{-1}\right] - \frac{3}{2}\mathrm{Tr}_V\left[N_0\Delta_{V0}^{-1}\right] \\
& - R\Big\{\frac{1}{2}\alpha_1\,\mathrm{Tr}_T\left[N\Delta_{T\alpha_2}^{-2}\right] + \frac{1}{2}\beta_1\,\mathrm{Tr}_T\left[N\Delta_{T\beta_2}^{-2}\right] - \frac{1}{2}\left(\alpha_1 - \frac{5}{12}\right)\mathrm{Tr}_V\left[N\Delta_{V\alpha_2}^{-2}\right] \\
& - \frac{1}{2}\left(\beta_1 - \frac{5}{12}\right)\mathrm{Tr}_V\left[N\Delta_{V\beta_2}^{-2}\right] + \frac{1}{4}\mathrm{Tr}_S\left[N\Delta_{S0}^{-2}\right] + \frac{1}{2}\gamma_1\,\mathrm{Tr}_S\left[N\Delta_{S\gamma_2}^{-2}\right] \\
& + \frac{1}{2}\delta_1\,\mathrm{Tr}_S\left[N\Delta_{S\delta_2}^{-2}\right] + \frac{1}{8}\mathrm{Tr}_V\left[N_0\Delta_{V0}^{-2}\right] - \left(\frac{1}{K-3} + \frac{1}{2}\right)\mathrm{Tr}_S\left[N_0\Delta_{S0}^{-2}\right]\Big\}.
\end{aligned} \tag{0.33}$$

The terms with N_0 are contributions of the ghosts.

As a next step we evaluate the traces. We use the heat kernel expansion which for an arbitrary function of the covariant Laplacian $W(D^2)$ reads

$$\mathrm{Tr}_j[W(-D^2)] = (4\pi)^{-2}\,\mathrm{tr}_j(I)\left\{Q_2[W]\int d^4x\sqrt{g} + \frac{1}{6}Q_1[W]\int d^4x\sqrt{g}R + 0(R^2)\right\} \tag{0.34}$$

where by I we denote the unit matrix in the space of fields on which D^2 acts. Therefore $\mathrm{tr}_j(I)$ simply counts the number of independent degrees of freedom of the field, namely

$$\mathrm{tr}_s(I) = 1, \qquad tr_V(I) = 4, \qquad tr_T(I) = 9.$$

The sort j of fields enters into (4.6) via $\mathrm{tr}_j(I)$ only. Therefore we will drop the index j of Δ_{ja} after evaluation of the traces in the heat kernel expansion.

The functionals Q_n are Mellin transforms of W:

$$Q_0[W] = W[0], \qquad Q_n[W] = \frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1}W(z) \qquad (n>0). \tag{0.35}$$

Now we have to perform the heat kernel expansion (4.10) in Eq. (4.11). This leads to a polynomial in R which is the r.h.s. of the evolution equation. By comparison of coefficients with the l.h.s. of the evolution equation (2.6) we obtain in the order of $\int d^4x\sqrt{g}$

$$\begin{aligned}
\partial_t[Z_{Nk}\Lambda_k] = {} & \frac{1}{8\kappa^2}\frac{1}{(4\pi)^2}\Big\{5Q_2\left[N\Delta_{\alpha_2}^{-1}\right] + 5Q_2\left[N\Delta_{\beta_2}^{-1}\right] + 7Q_2\left[N\Delta_0^{-1}\right] \\
& + Q_2\left[N\Delta_{\gamma_2}^{-1}\right] + Q_2\left[N\Delta_{\delta_2}^{-1}\right] - 6Q_2\left[N_0\Delta_0^{-1}\right]\Big\}
\end{aligned} \tag{0.36}$$

And in the order of $\int d^4x\sqrt{g}R$

$$\partial_t(Z_{Nk}) = -\frac{1}{24\kappa^2}\frac{1}{(4\pi)^2}\Big\{5Q_1\left[N\Delta_{\alpha_2}^{-1}\right] + 5Q_1\left[N\Delta_{\beta_2}^{-1}\right] + 7Q_1\left[N\Delta_0^{-1}\right] + Q_1\left[N\Delta_{\gamma_2}^{-1}\right] + Q_1\left[N\Delta_{\delta_2}^{-1}\right] - 12Q_1\left[N_0\Delta_0^{-1}\right] - 30\left(\alpha_1 + \frac{1}{12}\right)Q_2\left[N\Delta_{\alpha_2}^{-2}\right] - 30\left(\beta_1 + \frac{1}{12}\right)Q_2\left[N\Delta_{\beta_2}^{-2}\right] - 3Q_2\left[N\Delta_0^{-2}\right] - 6\gamma_1 Q_2\left[N\Delta_{\gamma_2}^{-2}\right] - 6\delta_1 Q_2\left[N\Delta_{\delta_2}^{-2}\right] + \frac{12}{K-3}Q_2\left[N_0\Delta_0^{-2}\right]\Big\}. \tag{0.37}$$

Let us introduce the cut-off-dependent integrals

$$\Phi_n^p(w) = \frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1}\frac{R^{(0)}(z) - zR^{(0)'}(z)}{[z + R^{(0)}(z) + w]^p},$$
$$\tilde{\Phi}_n^p(w) = \frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1}\frac{R^{(0)}(z)}{[z + R^{(0)}(z) + w]^p} \tag{0.38}$$

for $n > 0$. It follows that $\Phi_0^p(w) = \tilde{\Phi}_0^p(w) = (1+w)^{-p}$ for $n = 0$. In addition, we use the fact that

$$N = \frac{\partial_t\left[Z_{Nk}k^2R^{(0)}(-D^2/k^2)\right]}{Z_{Nk}} = [2 - \eta_N(k)]\,k^2R^{(0)}(z) + 2D^2R^{(0)'}(z) \tag{0.39}$$

with $\eta_N(k) = -\partial_t(\ln Z_{Nk})$ being the anomalous dimension of the operator $\sqrt{g}R$. Then we can rewrite Eqs. (4.8) and (4.9) in terms of Φ and $\tilde{\Phi}$. This leads to the following set of equations:

$$\partial_t[Z_{Nk}\Lambda_k] = \frac{1}{4\kappa^2}\frac{1}{(4\pi)^2}k^4\Big\{5\Phi_2^1(\alpha_{2k}) + 5\Phi_2^1(\beta_{2k}) - 5\Phi_2^1(0) + \Phi_2^1(\gamma_{2k}) + \Phi_2^1(\delta_{2k}) - \frac{1}{2}\eta_N(k)\left[5\tilde{\Phi}_2^1(\alpha_{2k}) + 5\tilde{\Phi}_2^1(\beta_{2k}) + 7\tilde{\Phi}_2^1(0) + \tilde{\Phi}_2^1(\gamma_{2k}) + \tilde{\Phi}_2^1(\delta_{2k})\right]\Big\}, \tag{0.40}$$

$$\partial_t(Z_{Nk}) = -\frac{1}{24\kappa^2}\frac{1}{(4\pi)^2}k^2\Big\{10\Phi_1^1(\alpha_{2k}) + 10\Phi_1^1(\beta_{2k}) - 10\Phi_1^1(0) + 2\Phi_1^1(\gamma_{2k}) + 2\Phi_1^1(\delta_{2k}) - (60\alpha_1 + 5)\Phi_2^2(\alpha_{2k}) - (60\beta_1 + 5)\Phi_2^2(\beta_{2k}) + \left(\frac{24}{K-3} - 6\right)\Phi_2^2(0) - 12\gamma_1\Phi_2^2(\gamma_{2k}) - 12\delta_1\Phi_2^2(\delta_{2k}) - \eta_N(k)\Big[5\tilde{\Phi}_1^1(\alpha_{2k}) + 5\tilde{\Phi}_1^1(\beta_{2k}) + 7\tilde{\Phi}_1^1(0) + \tilde{\Phi}_1^1(\gamma_{2k}) + \tilde{\Phi}_1^1(\delta_{2k}) - 30(\alpha_1 + \tfrac{1}{12})\tilde{\Phi}_2^2(\alpha_{2k}) - 30(\beta_1 + \tfrac{1}{12})\tilde{\Phi}_2^2(\beta_{2k}) - 3\tilde{\Phi}_2^2(0) - 6\gamma_1\tilde{\Phi}_2^2(\gamma_{2k}) - 6\delta_1\tilde{\Phi}_2^2(\delta_{2k})\Big]\Big\} \tag{0.41}$$

with

$$\alpha_{2k} = \alpha_2/k^2 = f^2\kappa_k^2 + f^2\kappa_k^2\left(1 + \frac{4\lambda_k}{Kf^2\kappa_k^2}\right)^{1/2},$$
$$\beta_{2k} = \beta_2/k^2 = f^2\kappa_k^2 - f^2\kappa_k^2\left(1 + \frac{4\lambda_k}{Kf^2\kappa_k^2}\right)^{1/2},$$
$$\gamma_{2k} = \gamma_2/k^2 = \nu^2\kappa_k^2 + \nu^2\kappa_k^2\left(1 - \frac{8\lambda_k}{(K-3)\nu^2\kappa_k^2}\right)^{1/2},$$
$$\delta_{2k} = \delta_2/k^2 = \nu^2\kappa_k^2 - \nu^2\kappa_k^2\left(1 - \frac{8\lambda_k}{(K-3)\nu^2\kappa_k^2}\right)^{1/2}, \tag{0.42}$$

where $\kappa_k^2 = \kappa^2/k^2$ and $\lambda_k = \Lambda/k^2$ and we have replaced the values of m_2^2 and m_0^2 in Eqs. (4.4) and (4.5), respectively. Now we introduce the dimensionless, renormalized Newtonian constant

$$g_k = k^2G_k = k^2Z_{Nk}^{-1}\bar{G}. \tag{0.43}$$

Here G_k is the renormalized Newtonian constant at scale k. The evolution equation for g_k then reads

$$\partial_t g_k = [2 + \eta_N(k)]\,g_k. \tag{0.44}$$

¿From (4.18) we find the anomalous dimension $\eta_N(k)$:

$$\eta_N(k) = g_k B_1(\kappa_k, \lambda_k) + \eta_N(k) g_k B_2(\kappa_k, \lambda_k) \tag{0.45}$$

where

$$\begin{aligned} B_1(\kappa_k, \lambda_k) = \frac{1}{12\pi}\Big\{ & 10\Phi_1^1(\alpha_{2k}) + 10\Phi_1^1(\beta_{2k}) - 10\Phi_1^1(0) + 2\Phi_1^1(\gamma_{2k}) + 2\Phi_1^1(\delta_{2k}) - (60\alpha_1 + 5)\Phi_2^2(\alpha_{2k}) \\ & - (60\beta_1 + 5)\Phi_2^2(\beta_{2k}) + \left(\frac{24}{K-3} - 6\right)\Phi_2^2(0) - 12\gamma_1\Phi_2^2(\gamma_{2k}) - 12\delta_1\Phi_2^2(\delta_{2k})\Big\}, \\ B_2(\kappa_k, \lambda_k) = -\frac{1}{12\pi}\Big\{ & 5\tilde{\Phi}_1^1(\alpha_{2k}) + 5\tilde{\Phi}_1^1(\beta_{2k}) + 7\tilde{\Phi}_1^1(0) + \tilde{\Phi}_1^1(\gamma_{2k}) + \tilde{\Phi}_1^1(\delta_{2k}) - 30(\alpha_1 + \tfrac{1}{12})\tilde{\Phi}_2^2(\alpha_{2k}) \\ & - 30(\beta_1 + \tfrac{1}{12})\tilde{\Phi}_2^2(\beta_{2k}) - 3\tilde{\Phi}_2^2(0) - 6\gamma_1\tilde{\Phi}_2^2(\gamma_{2k}) - 6\delta_1\tilde{\Phi}_2^2(\delta_{2k})\Big\}. \end{aligned} \tag{0.46}$$

Solving (4.21),

$$\eta_N(k) = \frac{g_k B_1(\kappa_k, \lambda_k)}{1 - g_k B_2(\kappa_k, \lambda_k)}, \tag{0.47}$$

we see that the anomalous dimension η_N is a nonpertubative quantity. From (4.16) we obtain the evolution equation for the cosmological constant

$$\begin{aligned} \partial_t(\lambda_k) = -\left[2 - \eta_N(k)\right]\lambda_k + \frac{g_k}{4\pi}\Big\{ & 10\Phi_2^1(\alpha_{2k}) + 10\Phi_2^1(\beta_{2k}) - 10\Phi_2^1(0) + 2\Phi_2^1(\gamma_{2k}) + 2\Phi_2^1(\delta_{2k}) \\ & - \eta_N(k)\left[5\tilde{\Phi}_2^1(\alpha_{2k}) + 5\tilde{\Phi}_2^1(\beta_{2k}) + 7\tilde{\Phi}_2^1(0) + \tilde{\Phi}_2^1(\gamma_{2k}) + \tilde{\Phi}_2^1(\delta_{2k})\right]\Big\}. \end{aligned} \tag{0.48}$$

Eqs. (4.20) and (4.24) with (4.23) determine the value of the running Newtonian constant and cosmological constant at the scale $k << \Lambda_{\text{cut-off}}$. The above evolution equations include nonperturbative effects which go beyond a simple one-loop calculation. This is particularly obvious if one expands (4.23) for small values of g_k:

$$\eta_N = g_k B_1(\kappa_k, \lambda_k)\Big[1 + g_k B_2(\kappa_k, \lambda_k) + g_k^2 B_2^2(\kappa_k, \lambda_k) + ...\Big]. \tag{0.49}$$

estartsectionsection10pt-3.5ex plus -1ex minus -.2ex2.3ex plus .2exCritical points and the running Newtonian coupling constant

In the present section we give some remarks about the properties of nonperturbative RG equations. First of all, let us estimate the qualitative behaviour of the running gravitational coupling constant.

The dimensional quantity G_k evolves according to

$$\partial_t G_k = \eta_N G_k \tag{0.50}$$

The set of RG equations for the coupling constants is too complicated and cannot be solved analytically. Hence we assume that the cosmological constant is much smaller than the IR cut-off scale, $\lambda_k << k^2$, so we can put $\lambda_k \sim 0$. This simplifies Eqs. (4.29), (4.24) and we are left with only Eq. (5.1). After that we perform an expansion in powers of $(\bar{G}_k^2)^{-1}$ keeping only the first term (i.e. we evaluate the functions $\Phi_n^p(0)$ and $\bar{\Phi}_n^p(0)$) and finally obtain (with $g_k \sim k^2\bar{G}$)

$$G_k = G_o\left[1 - w\bar{G}k^2 + ...\right] \tag{0.51}$$

where

$$w = -\frac{1}{2}B_1(0,0) = \frac{1}{24\pi}\left[\left(50 + 22\frac{f^2}{\nu^2}\right) - \frac{7\pi^2}{3}\right].$$

In obtaining w we use the same cut-off function as in Ref. [8].

For Einstein gravity in the same formalism (also using the gauge-fixing independent EA) we have got [9]:

$$w = \frac{\pi}{36}\left[\frac{108}{\pi^2} - 1\right] \tag{0.52}$$

In the case under discussion we see that the sign of w depends on the higher-derivative coupling constants:

$$w > 0 \qquad \text{if} \qquad 50 - \frac{7\pi^2}{3} + \frac{22f^2}{\nu^2} > 0. \tag{0.53}$$

The coupling constant ν^2 maybe chosen to be negative (see [2]). So, e.g., for $f^2 = 1$, $\nu^2 = \pm 1$ we get $w > 0$ and the Newtonian coupling decreases as k^2 increases. In other words, we find an antiscreening behaviour of the gravitational coupling. On the contrary, for $f^2 = 1$, $\nu^2 = -1/2$ we get $w < 0$ and a screening behaviour for the Newtonian coupling (for the one-loop behaviour of the Newtonian coupling in R^2-gravity with matter, see [19]). This means that in such a phase the gravitational charge (mass) is screened by quantum fluctuations, or, in other words, the Newtonian coupling is smaller at smaller distances. The sign of a quantum correction to the Newtonian potential will be different too. Note that our solution (5.2) is actually qualitative, and the full RG system should be analyzed for a better result.

Our main qualitative result is that R^2-gravity considered as an effective theory may change the low-energy gravitational phenomena as compared with Einstein gravity.

Let us now investigate the problem of existence of critical points in the theory under study. We search for points at which the r.h.s. of Eqs. (4.20) and (4.24) are equal to zero (supposing $g_k = k^2\bar{G}$). For Einstein gravity such a study may be carried out quite easily (since the functions B_1 and B_2 depend only on λ_k). A numerical analysis of the corresponding RG system which is written in the physical Landau-DeWitt gauge gives Ref. [9]:

$$\lambda_k = 0.352, \qquad g_k = 0.348. \tag{0.54}$$

These points actually correspond to UV-stable fixed points. Note that the solutions (5.5) do not give a solution to the cosmological constant problem as a result of the non-perturbative RG behaviour.

In R^2-gravity the situation is much more complicated because the functions B_1 and B_2 depend on κ_k and λ_k and because there are the higher-derivative coupling constants as free parameters of the theory. Supposing $g_k = k^2\bar{G}$, we can get the following unstable fixed points for $f^2 = 1/10$ and $\nu^2 = -1/4$:

$$\lambda_k = 4.47, \qquad g_k = 4.46. \tag{0.55}$$

Note that for other values of the higher-derivative coupling constants one can get numerically other values for the unstable fixed points.

estartsectionsection10pt-3.5ex plus -1ex minus -.2ex2.3ex plus .2exDiscussion

In the present work we have studied the truncated evolution equation in higher-derivative quantum gravity. Making a truncation to the space of low-derivative functionals, we have obtained nonperturbative RG equations for the Newtonian and cosmological coupling constants. The necessary step in such a study is the calculation of the effective average action on some background (we have used the de Sitter background).

The properties of the nonperturbative RG equations (like the existence of critical points and the behaviour of the Newtonian coupling) are discussed. As we showed, the higher-derivative QG may behave at low energies qualitatively different from Einstein QG.

The next open problem in such an approach is to make a better truncation of the evolution equation to the space of functionals with higher derivatives. In such a way one could obtain a complete set of nonperturbative RG equations for all coupling constants: f^2, ν^2, κ^2, Λ. Hence, unlike the present study where f^2 and ν^2 are free parameters, we might define the critical points of the complete phase space (the RG equations for κ^2 and Λ are, of course, the same).

However, in order to find the effective average action with a truncation to the space of higher-derivative functionals, we have to perform a calculation of the one-loop effective action in a background where R^2 and $C^2_{\mu\nu\alpha\beta}$ may be distinguished. Clearly the de Sitter space does not belong to this class of backgrounds.

As far as we know (see [2] for a review), the one-loop effective action for R^2-gravity has been found only in the de Sitter or flat backgrounds. Even such a calculation is extremely complicated. A generalization of such a result to a more complicated background (say, of the above sort) being, in principle, possible, is extremely complicated. Moreover, that is just one step in writing the r.h.s. of the evolution equation. After that, much more work is required to obtain explicitly the nonperturbative RG equations for ν^2 and f^2. Hence, this problem is left for future research.

Another related problem is the gauge dependence of the average effective action. In order to solve this problem, one has to do an even better truncation which includes all gauge parameters as independent functions of k. Hence, in addition to the four RG equations for the coupling constants, one should write some RG equations for all gauge parameters. Then the RG equations for the gauge parameters should lead to some stable fixed points. These fixed point values for the gauge parameters should be used in the RG equations for the coupling constants. It is clear that such a programme is too complicated and cannot be realized.

However, there is a simpler way which we have actually used in this work. In a study of the effective average action for the Yang-Mills theory (see [20]) a k-dependent gauge parameter was used. It has been shown that there exists an attracting fixed point of the truncated evolution for the gauge parameter. This fixed point corresponds to the so-called Landau-DeWitt gauge. Hence, the alternative easy way of studying the truncated evolution equation is to work in the physical Landau-DeWitt gauge (actually it corresponds to a study in the gauge-fixing-independent effective action formalism). Similarly in Einstein gravity, in order to avoid the introduction of a tedious additional RG equation for the gauge parameter, one

can work in the Landau-DeWitt gauge which again corresponds to the gauge-fixing-independent EA formalism (see [9]). In the same way, we have used here the gauge-fixing-independent EA in order to solve the gauge dependence problem for R^2-gravity in our formulation.

Hence, our study which indicates the qualitative difference between Einstein and R^2-gravity even at low energy scales is a necessary step in the formulation of better truncations of the evolution equation in R^2-gravity. Moreover, it is expected to be useful also in the studies of supersymmetric R^2-gravity in a nonperturbative approach.

estartsectionsubsection20pt-3ex plus -1ex minus -.2ex1.4ex plus .2ex*Acknowledgement We would like to thank A.A. Bytsenko for helpful discussions and participation in part of this work and A. Romeo for help in numerical calculations. We are grateful to M. Reuter for useful e-mail discussions. L.N.G. was supported by COLCIENCIAS (Colombia) Project No. 1106-05-393-95. S.D.O was supported in part by COLCIENCIAS.

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